Sextant sight reduction with AstronavXls software (ver. 13.3)

1. Introduction

This software for sextant astronomical sight reduction is based on a **Microsoft Office Excel workbook**, using **macro-commands**. It was developed to help solving sextant sight data quickly and without using correction tables other than the daily pages of the *Nautical Almanac*.

Through celestial (or terrestrial) sphere projection graphs, it allows visualizing the parameters of the observed celestial body and of the observer's position. A special worksheet is dedicated to the plotting of the reduced data (circles of position and azimuth lines) in order to establish the observer's position from the sights. The visual approach to illustrate the different sight reduction steps has a didactic purpose and the software could be used as a tool to complement a tutorial on astronomical navigation and sextant sight reduction.

Following that line of thought, this software can be considered more as a perfectible teaching aid and may contain errors affecting navigation. Hence, there is no warranty pertaining to the accuracy and the results obtained from using the product. However, verifications performed to date yield results quite comparable to those obtained with traditional forms and methods of astronomical calculation.

A brief description of the software characteristics follows.

2. Software structure

The **AstronavXls** software consists of a **macro-enabled Excel workbook** of type ***.xlsm** (Office 2013), containing the necessary formulas and macros pertinent to astronomical sight reduction. In order for the software to operate properly, the **macro option** must be enabled in the worksheets, using the following procedure:

– Click on the "File" tab (upper left corner), then select "Options / Trust Center / Trust center Settings / Macro Settings / Enable all macros".

– The procedure may be different in older Excel versions.

There are three types of worksheets: the **Sight worksheets**, numbering three, the **LOP plot worksheet**, and the **CALC worksheets**, also numbering three, one for each Sight worksheet.

The CALC worksheets contain the data and calculations for the graphs in each Sight worksheet and for the LOP plot worksheet. They are for reference only and the user should not enter or modify any data on those sheets. They are hidden and will not be detailed in this document. They can be shown by right-clicking on one of the sheet tabs at the bottom of the worksheet, and selecting the name of the sheet to be displayed.

All the sheets are protected (no password) in order to avoid inadvertently modifying essential formulas or data. Only the cells used for data inputs can be modified. However, if required, the user can manually unprotect the sheets and make the modifications desired.

The following sections will discuss in more details the input data and functions in the "Sight" and "LOP plot" worksheets.

3. The Sight Worksheets

The Sight worksheets are divided in three sections: a **data input section**, a **celestial sphere projection section**, showing graphs of different projection of the celestial or terrestrial sphere, and a **calculator utilities section**. A brief description of some parameters appears on the right side of the different data input sections, and a small red flag in the upper right corner of the description cell indicates that supplementary information is given as "comment" attached to the cell.

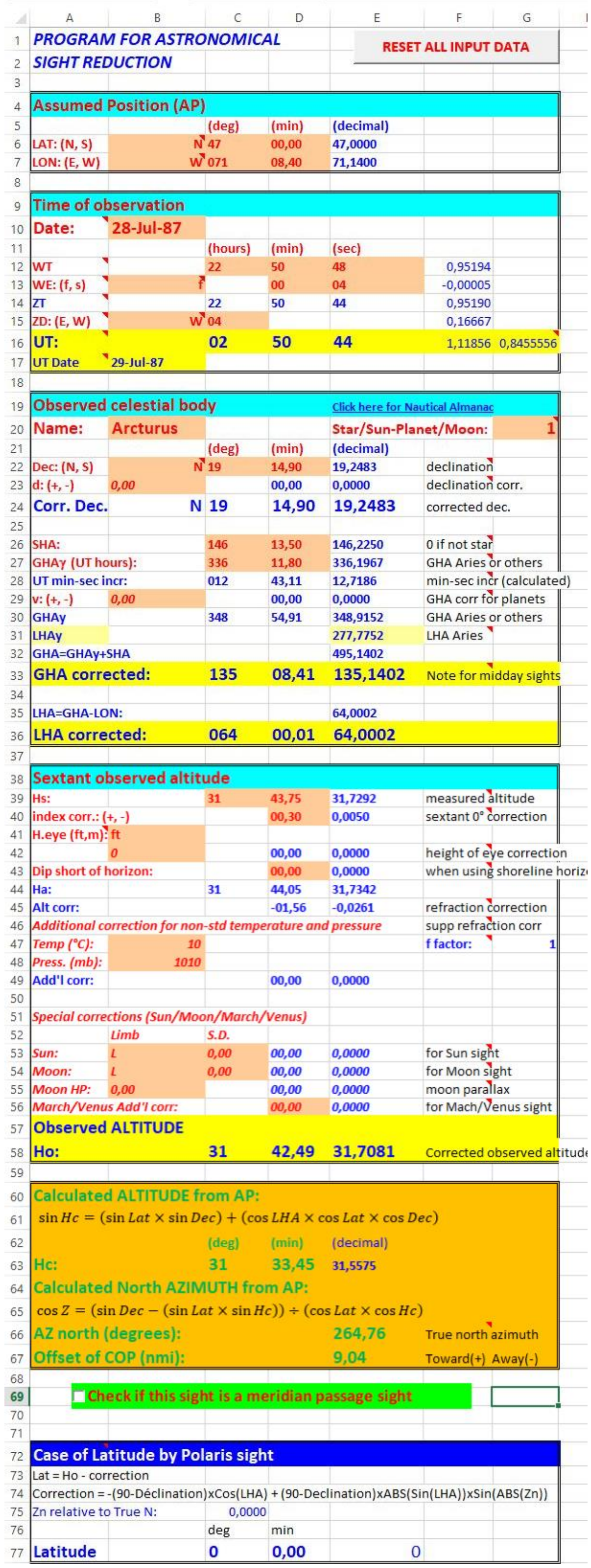
3.01. The data input section

This section, on the left side of the worksheet, (Figure 1) contains all the necessary parameters pertinent to the celestial body observation and the astronomic reduction of the sight. The data to be entered by the user are indicated in red, while the calculated values appear in blue.

The Data input section is subdivided into four input blocks (on green background): **Assumed Position (AP)**, **Time**, **Observed body** and **Sextant observed Altitude**. The input blocks are followed by a block of **calculated results**, which are the calculated altitude (Hc) of the celestial body at the Assumed Position, and the North Azimuth (AZ North) toward that celestial body. The angular values are entered in degrees-minutes.decimal format and a column to the right indicates the corresponding degrees.decimal values. The Watch Time (WT) of the observation is entered in hours-minutes-seconds.

The parameters and functions appearing in the Sight worksheet / Data input section are described below.

Reset button: this button resets all sight input data to 0 or default values; it



	A	B	C	D	E	F	G
1	PROGRAM FOR ASTRONOMICAL					RESET ALL INPUT DATA	
2	SIGHT REDUCTION						
3							
4	Assumed Position (AP)						
5			(deg)	(min)	(decimal)		
6	LAT: (N, S)	N	47	00,00	47,0000		
7	LON: (E, W)	W	071	08,40	71,1400		
8							
9	Time of observation						
10	Date:	28-Jul-87					
11			(hours)	(min)	(sec)		
12	WT		22	50	48	0,95194	
13	WE: (f, s)	f		00	04	-0,00005	
14	ZT		22	50	44	0,95190	
15	ZD: (E, W)	W	04			0,16667	
16	UT:		02	50	44	1,11856	0,8455556
17	UT Date	29-Jul-87					
18							
19	Observed celestial body				Click here for Nautical Almanac		
20	Name:	Arcturus			Star/Sun-Planet/Moon:		1
21			(deg)	(min)	(decimal)		
22	Dec: (N, S)	N	19	14,90	19,2483	declination	
23	d: (+, -)	0,00		00,00	0,0000	declination corr.	
24	Corr. Dec.	N	19	14,90	19,2483	corrected dec.	
25							
26	SHA:		146	13,50	146,2250	0 if not star	
27	GHAγ (UT hours):		336	11,80	336,1967	GHA Aries or others	
28	UT min-sec incr:		012	43,11	12,7186	min-sec incr (calculated)	
29	v: (+, -)	0,00		00,00	0,0000	GHA corr for planets	
30	GHAγ		348	54,91	348,9152	GHA Aries or others	
31	LHAγ				277,7752	LHA Aries	
32	GHA=GHAγ+SHA				495,1402		
33	GHA corrected:		135	08,41	135,1402	Note for midday sights	
34							
35	LHA=GHA-LON:				64,0002		
36	LHA corrected:		064	00,01	64,0002		
37							
38	Sextant observed altitude						
39	Hs:		31	43,75	31,7292	measured altitude	
40	index corr.: (+, -)			00,30	0,0050	sextant 0° correction	
41	H.eye (ft,m):	ft					
42		0		00,00	0,0000	height of eye correction	
43	Dip short of horizon:			00,00	0,0000	when using shoreline horiz	
44	Ha:		31	44,05	31,7342		
45	Alt corr:			-01,56	-0,0261	refraction correction	
46	Additional correction for non-std temperature and pressure					supp refraction corr	
47	Temp (°C):	10				f factor:	1
48	Press. (mb):	1010					
49	Add'l corr:			00,00	0,0000		
50							
51	Special corrections (Sun/Moon/March/Venus)						
52		Limb	S.D.				
53	Sun:	L	0,00	00,00	0,0000	for Sun sight	
54	Moon:	L	0,00	00,00	0,0000	for Moon sight	
55	Moon HP:	0,00		00,00	0,0000	moon parallax	
56	March/Venus Add'l corr:			00,00	0,0000	for Mach/Venus sight	
57	Observed ALTITUDE						
58	Ho:		31	42,49	31,7081	Corrected observed altitude	
59							
60	Calculated ALTITUDE from AP:						
61	$\sin Hc = (\sin Lat \times \sin Dec) + (\cos LHA \times \cos Lat \times \cos Dec)$						
62			(deg)	(min)	(decimal)		
63	Hc:		31	33,45	31,5575		
64	Calculated North AZIMUTH from AP:						
65	$\cos Z = (\sin Dec - (\sin Lat \times \sin Hc)) \div (\cos Lat \times \cos Hc)$						
66	AZ north (degrees):				264,76	True north azimuth	
67	Offset of COP (nmi):				9,04	Toward(+) Away(-)	
68							
69		Check if this sight is a meridian passage sight					
70							
71							
72	Case of Latitude by Polaris sight						
73	Lat = Ho - correction						
74	Correction = -(90-Déclination)xCos(LHA) + (90-Declination)xABS(Sin(LHA))xSin(ABS(Zn))						
75	Zn relative to True N:		0,0000				
76			deg	min			
77	Latitude		0	0,00	0		

Figure 1. Data input section of the Sight worksheet.

also resets to 0 the parameters of the running circles of position (COP) and erases the corresponding best fit tangential linear estimates parameters, if any, in the LOP plot worksheet.

3.01.01. The Assumed Position (AP) block:

Latitude and Longitude: the assumed latitude (LAT) and longitude (LON), closest to the observer's actual position, are entered here. Enter also in the adjacent cells **N** or **S** for North (+) or South (-) latitude, and **E** or **W** for East (-) or West (+) longitude. The program will select automatically the proper sign. The computed altitude (Hc) of the observed celestial body and its azimuth are calculated from this assumed position (AP). The azimuth line to the body, as well as the AP, serve as references to plot the intercept of the circle of position (COP), according to the difference between the observed corrected altitude (Ho) and the computed altitude (Hc).

3.01.02. The Time block:

Date: enter the date corresponding to the Watch Time (WT) of the observation, or the local Zone Time (ZT).

Watch Time (WT): enter the time of the observation as read on the watch used on the boat for sextant sights. Normally set at the local Zone Time (ZT).

Watch Error (WE): enter the cumulative error of the watch with respect to the Coordinated Universal Time (UT); if the watch is slow: enter **s** (+); if fast: enter **f** (-). The sign is taken care of automatically.

Zone Time (ZT): the time of the time Zone in which you happen to be, or which you are using on the boat. This is the corrected watch time.

Zone Description (ZD): enter the number representing the time Zone (15° wide) in which you are, or that you are using on the boat. It corresponds to the integer value of your longitude divided by 15 (bottom-rounded if the decimal is less than 0.5). If the decimal is equal or more than 0.5, use the top-rounded integer value. You must also enter in the adjacent cell **E** or **W** depending if your position is East (-) or West (+) of Greenwich. The program will select automatically the proper sign.

UT: the Universal Time (also Zulu time, Greenwich Mean Time or Coordinated Universal Time) of the observation, in hours-minutes-seconds. This value is calculated from the previous entries.

UT date: the date corresponding to the UT observation time. It may be different from the date of your watch time set at ZT. This value is calculated from the above time data.

3.01.03. The Observed celestial body block:

Name: enter the name of the observed celestial body.

Star/Sun-Planet/Moon: the number entered here is an indicator for the program to select the appropriate time base for the "minutes-seconds" increment interpolation to be added to the whole hour Greenwich Hour Angle (GHA) obtained from the *Nautical Almanac*. As an example, the sun and planets move 15° in one hour, as opposed to 15.04° for the stars and 14.32° for the moon. So, **it is very important to make the appropriate choice here**.

Declination: the declination (DEC) of a celestial body is its elevation above the projected equator on the celestial sphere. This value is obtained from the *Nautical Almanac*. You must also enter **N** or **S** to indicate North (+) or South (-) declination. The program will select automatically the proper sign.

d correction: the "d" correction (mainly for sun, moon and planets) is the hourly variation of the declination. It is used in conjunction with the UT minutes-seconds increment to interpolate the exact declination at the observation time, and is obtained from the *Nautical Almanac*. It is positive if the declination is hourly increasing (North or South), negative if hourly decreasing.

SHA: the sidereal hour angle (**for stars only**), obtained from the *Nautical Almanac*, is the westward angular position of a star meridian, measured along the projected equator and referenced to the position of the first point of Aries (the intersection of the equator and the ecliptic at the vernal equinox, symbolized by the Greek letter **γ**). The eastward angle is known as the Right Ascension (RA). If the observed body is not a star, enter 0 in this cell.

GHAγ (or other celestial bodies) for whole hour: the Greenwich hour angle of the first point of Aries (used to find the SHA of stars) corresponds to the position of the meridian of the first point of Aries measured westward along the projected equator, from the Greenwich meridian. This value is obtained from the *Nautical Almanac* for whole UT hour values. **For other celestial bodies** such as the Sun, the Moon or planets, **their GHA** is also obtained directly from the *Nautical Almanac* for whole UT hours, and **should be entered here** instead of the GHA of Aries.

UT min-sec increment of the GHA: this GHA increment is calculated automatically from the UT minute-second values of the observation time and added to the GHA value obtained from the *Nautical Almanac* for the whole hour UT value smaller than the actual time of observation. The hour-angle base to compute this GHA increment is selected according to the number entered under the **"Star/Sun-Planet/Moon" input**.

v correction: the "v" correction, mainly for planets and the moon, is an additional hourly variation of the body GHA accounting for non-uniform apparent motions of moon and planets. It is used in conjunction with the UT minute-second of the observation to correct the GHA at the observation time, and is obtained from the *Nautical Almanac*. It is positive if hourly increasing, negative if hourly decreasing.

LHAγ: the local hour angle of the first point of Aries corresponds to the hour angle of the meridian of the first point of Aries, westward from the observer's meridian. It also happens to be the right ascension of the observer's meridian (angle measured eastward from the first point of Aries), a value required to properly set a **star finder**. This is the only reason why this parameter is calculated here, as it is not used for any other purpose. Combined with the sextant altitude Hs of the body and its approximate azimuth (measured with the compass), this parameter allows to identify the observed body by mean of the star finder. It is obtained by:

$$LHA_{\gamma} = GHA_{\gamma} - LONGITUDE \text{ (west longitude = +)}$$

GHA: this is the Greenwich hour angle of the meridian of the celestial body at the time of observation, measured westward on the equator from the Greenwich meridian. For a star, it is obtained by:

$$GHA_{star} = GHA_{\gamma} + SHA$$

GHA corrected: sometimes, adding GHAγ and SHA may yield numbers >360°. The corrected GHA is always <360°.

LHA: the local hour angle, or the angular hour angle of the meridian of the observed body with respect to the observer's meridian. It is calculated as follows, using the sign convention:

$$LHA = GHA - LONGITUDE \text{ (west longitude = +)}$$

The time diagram illustrates this angle, an important parameter of the navigation spherical triangle to be solved.

LHA corrected: using the sign convention, sometimes the subtraction of the longitude from the GHA may yield numbers >360° or <0. The corrected LHA is always positive and <360°.

3.01.04. The Sextant observed altitude block:

Hs: the altitude above horizon of the observed celestial body, as read from the sextant scale index and vernier.

Index correction: the known index correction in the sextant reading. It is measured by bringing the reflected image of a star (or of the horizon) over itself, so that the reading should be 0. A non-zero reading is the index error and the correction can be positive (off scale) or negative (in scale).

Dip (height of eye (ft,m)): enter the height of the eye above water (in meters or in feet), to calculate the dip error when taking a sight. This correction is always negative and calculated in arc-minutes from the following equation [1]:

$$DIP = -0.97\sqrt{Height_{eye\,(ft)}} = -1.76\sqrt{Height_{eye\,(m)}}$$

Dip short of horizon: when using the shoreline as horizon, or some other object on water closer than the horizon, the dip short correction is negative and is calculated in minutes from:

$$DIP_{SH} = -60 \times \tan^{-1}\left(\frac{Height_{eye\,(ft)}}{6076.1 \times dist} + \frac{dist_{nmi}}{8268}\right) \quad \text{or}$$

$$DIP_{SH} = -60 \times \tan^{-1}\left(\frac{Height_{eye\,(m)}}{1852 \times dist} + \frac{dist_{nmi}}{8268}\right)$$

With $Height_{eye}$ in feet or meters and the distance to the dip short horizon, *dist*, in nautical miles (nmi).

The **dip-short-of-horizon calculator**, in the calculator section of the worksheet, **must be used** to obtain this value. The resulting value is entered in the appropriate cell by the user.

Ha: the apparent altitude is evaluated by adding the index and dip corrections to Hs.

Altitude correction: the altitude correction is a correction for the refraction effect in the atmosphere, under standard atmospheric conditions (an air temperature of 10°C and an atmospheric pressure of 1010 mbar). This correction is also negative as the refraction always makes the celestial body to appear higher than reality. The correction, in minutes, is calculated from the following Smart formula [2]:

$$Alt_{corr} = -\left(\frac{0.97127}{\tan(Ha)} - \frac{0.00137}{\tan^3(Ha)}\right) \text{ for } 15° \leq Ha \leq 90°$$

$$Alt_{corr} = -\left(\frac{34.133 + 4.197 \cdot Ha + 0.00428 \cdot Ha^2}{1 + 0.505 \cdot Ha + 0.0845 \cdot Ha^2}\right) \text{ for } 0° \leq Ha < 15°$$

Additional altitude correction: this additional altitude correction is a refraction supplementary correction calculated from the non-standard atmospheric temperature and pressure conditions entered in the corresponding cells. This correction is added to the above standard altitude correction. The temperature and pressure factor *f* is first evaluated [2]:

$$f = \left(\frac{p_{mbar}}{1010}\right) \times \left(\frac{283}{273 + T[°C]}\right)$$

Then the additional correction is calculated:

$$Add'l_{corr} = Alt\_corr \times (f - 1)$$

Special corrections /Sun /Moon /March /Venus: the Sun, the Moon, March and Venus necessitate special altitude corrections. Because of their size, the observer cannot use the geometrical center of the Sun and the Moon to make an altitude measurement; the upper or lower limb must be used, which necessitates a **correction for the semi-diameter (S.D.)** of those bodies. For the sun, the apparent semi-diameter varies because the earth orbit is not circular and the distance earth-sun varies. Similarly, the moon orbit around the earth is not circular. Both the Sun and the Moon also present **horizontal parallax (H.P.)** effects, caused by the earth radius. The horizontal parallax effect varies with the measured altitude and is greatest when the body is near the horizon. Depending on the calculation method, an **augmentation** correction may be required to account for the difference between the moon geocentric S.D. and topographic S.D. (as seen from the surface of the earth). Finally, March and Venus necessitate an additional correction because of their **phase aspect** (not being fully illuminated), which distorts their apparent center.

Sun: enter the limb used for the sight (U for upper or L for lower) and the geocentric semi-diameter of the sun (S.D.*sun* is obtained at the bottom of the Sun data column in the *Nautical Almanac*). The calculated correction (arc-minutes) includes the semi-diameter correction and the horizontal parallax effect according to the following equation:

$$Sun_{special\ corr} = (\pm_U^L) S.D._{sun} + 0.1467 \times \cos(Ha + Alt_{corr} + Add'l_{corr})$$

Moon: for the Moon, the first correction concerns the horizontal parallax, which converts the altitude measurement to a geocentric reference. It is calculated from the H.P. value obtained from the Moon data column of the *Nautical Almanac*, using the equation below:

$$Moon_{H.P.} = H.P. \times \cos(Ha + Alt_{corr} + Add'l_{corr})$$

Next, enter the limb of the moon (U or L) used for the altitude measurement and the geocentric semi-diameter of the moon (S.D.*moon* is obtained from the Moon data column of the *Nautical Almanac*). Since after the horizontal parallax correction the altitude is geocentrically referenced, there is no need for the augmentation correction.

$$Moon_{special\ corr} = (\pm_U^L)(S.D._{moon})$$

Ho: the observed altitude, in degrees, is obtained by adding all the pertinent corrections to the apparent altitude (Ha). The difference between Ho and the computed altitude (Hc) will be used to evaluate the **intercept** of the circle of position with respect to the assumed position (AP), on the azimuth line.

Hc: the computed altitude, in degrees, is obtained from the entered AP data and the celestial body data, using the formula below:

$$Hc = \sin^{-1}(\sin(LAT) \times \sin(DEC) + \cos(LAT) \times \cos(DEC) \times \cos(LHA))$$

AZ: the azimuth direction of the celestial body from the AP is calculated from the following equation:

$$AZ = \cos^{-1}\left(\frac{\sin(DEC) - \sin(LAT) \times \sin(Hc)}{\cos(LAT) \times \cos(Hc)}\right)$$

This formula gives what is called the altitude azimuth, as it is a function of altitude parameters only, and it has an ambiguity for the determination of the direction (clockwise or anticlockwise) of the calculated angle with respect to true North.

AZ_North: the azimuth North value is the true North azimuth, where the ambiguity has been removed using the LHA value as follows:

$$IF\ LHA > 180°;\ AZ_{NORTH} = 360° - AZ$$

The values of **Ho**, **Hc** and **AZ_North** are the three navigation parameters necessary to plot a circular line of position in the LOP plot worksheet. An azimuth line is drawn from the assumed position AP. Then if the intercept difference between Ho and Hc (ΔH = Ho – Hc, converted to minutes of arc) is positive, a circle of position (COP) is drawn perpendicular to the azimuth line and intersecting it some ΔH nautical miles toward the celestial body, from the AP. If ΔH is negative, the COP is drawn away from the celestial body, from the AP.

3.01.05. The meridian passage sight checkbox (meridian transit)

This green checkbox allows to process meridian passage sights, or midday sights (sun sight case)**.**

In practice, meridian passage sights are used mainly to obtain the latitude of the observer because the longitude information from the time of transit, or the peak altitude of the body, is not accurate, the body appearing to hang for a while at its maximum altitude. A meridian passage sight may be used in conjunction with other more standard sights to provide a full solution of the position.

However, many years before the appearance of GPS technologies, surveyors obtained very accurate astronomical positions by measuring the meridian transit of a number of stars by means of a T-4 theodolite, and averaging the calculated coordinates. It is possible to use a similar technique with AstronavXls, but with a reduced number of sights.

The meridian passage checkbox appears on all Sight worksheets, and the result for each sight as well as the average coordinates for up to 3 sights (for the same position) are calculated in the green area of the **LOP plot** sheet. Checking the midday sight checkbox on the Sight sheet transfers the **corrected GHA** value as the **AP (DR) Longitude entry**, after conversion to equivalent East or West longitude. This transfer will bring the azimuth line of the celestial body to coincide with the meridian at the time of the sight. The latitude is found at the intersection of the circle of position with the azimuth line. Those coordinates, identified as the **Position of COP1** in the **LOP plot sheet,** or **PLOP1 on the graph**, correspond then to the **actual position of the observer at the meridian passage** of the celestial body. The coordinates of this position are converted to North/South latitude and East/West longitude in the **green area** to the right of the graph on the **LOP plot** sheet, labeled **Meridian passage sight position.**

Now, usually, the sun or the moon are the main celestial bodies used for meridian transit sights. But, for those who just want to experiment with their sextant in their backyard, it is possible to perform star meridian transit measurements all night, using an artificial horizon made out of a leveled bathroom mirror (there are high precision levels to 2 seconds of arc), and averaging the results to improve the accuracy of the longitude coordinate. The check box to the right of the **green area** allows to calculate the average of up to three transits measured at a same position.

3.01.06. The Latitude by Polaris sight block

Using Polaris (the Pole star) to measure Latitude is a well-known methodology; in short, the Latitude of the observer is more or less the same as the altitude of the pole star above the horizon. The *Nautical Almanac* contains the *Polaris Tables* (p.276) giving corrections factors (a_0, a_1 *and* a_2), functions of the *LHA of Aries*, the Latitude and the month of the year. These corrections are necessary due to the fact that Polaris is not directly over the North Pole (NP),

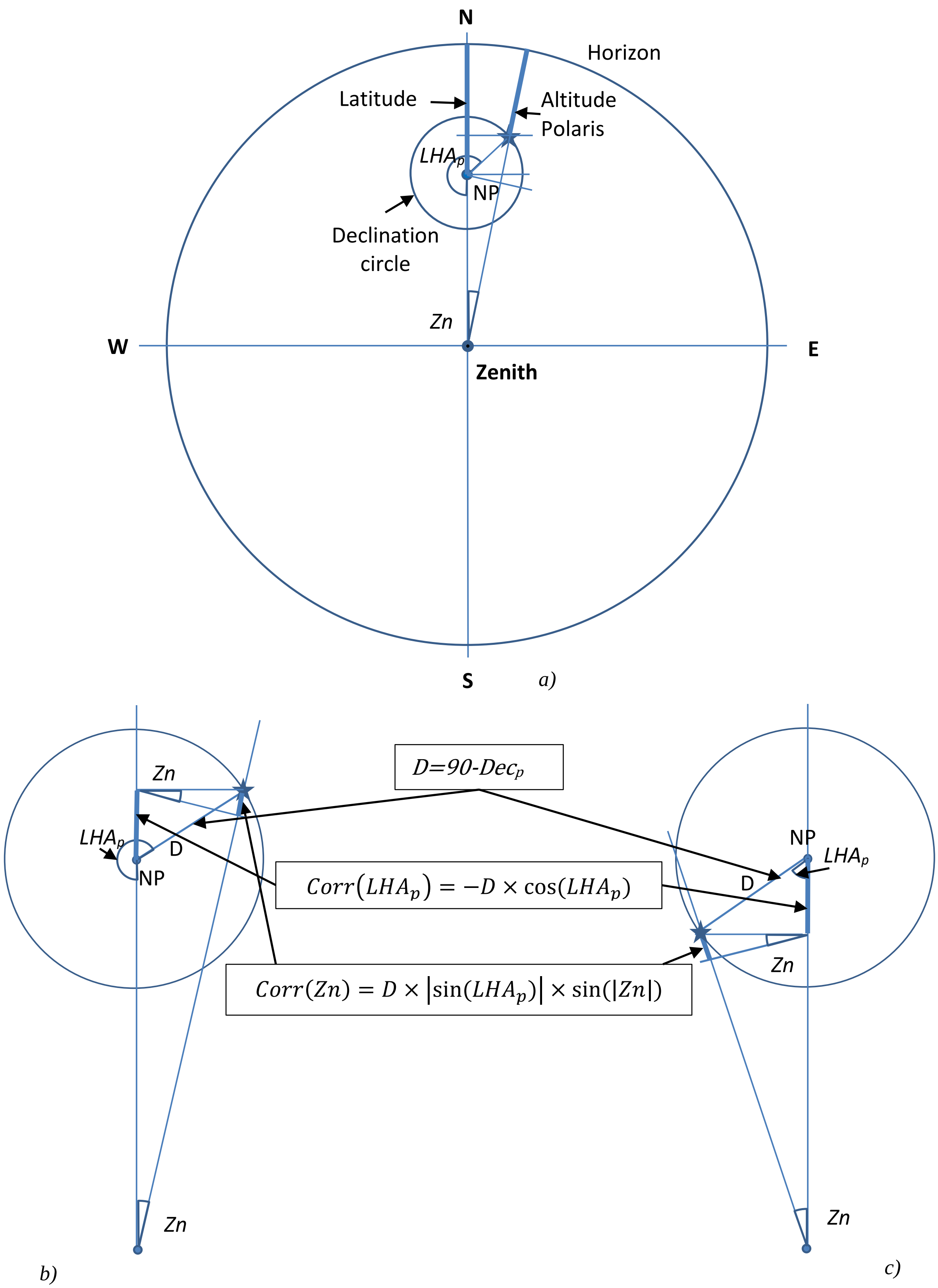


Figure 2. Calculating the corrections for a Latitude measurement from a Polaris sight.

but is rather offset by about 1° (the angle varies over one year), and therefore appears to turn around the Pole with the rotation of the earth, like any other star.

However, nowadays the *Nautical Almanac* also list Polaris together with the other navigation stars, with its *SHA* and *Declination*. Hence, it is possible to do the sight reduction calculations like any other star, however small the angles may be. The Latitude may also be calculated in a way similar to the good old procedure, but instead of using the correction factors as listed in the Polaris Tables, the correction can be calculated using the *SHA* and *Declination* data.

Figure 2 a) illustrates a view of an observer's celestial hemisphere, as he is taking a sextant measurement of the altitude of Polaris above the horizon. In this diagram, moving from the horizon circle towards its center (zenith position) means increasing altitude angle, up to the zenith of the observer. As can be seen, because of the offset of Polaris with respect to the North Pole, the azimuth (Zn) of the star is different from true North and can be on either side of true North, positive (clockwise) or negative (anticlockwise). The real altitude of the true North Pole above the horizon, if it could be measured directly, would give the Latitude of the observer (the darker line from horizon to North Pole). However, we can only measure the elevation of Polaris along its azimuth line, and since it is offset to the North Pole, corrections must be applied to find the observer's Latitude.

Figure 2 b) and c) are a magnification of the geometric figures, with the azimuth of Polaris on either side of true North, below or above the North Pole, illustrating the projection schemes for the corrections calculations.

The LHA_p of Polaris is found from its Almanac SHA_p value:

$$LHA_p = SHA_p + GHA_\gamma \begin{matrix} - Longitude\ W \\ + Longitude\ E \end{matrix}$$

Then, the distance *D* between Polaris and the true North is calculated:

$$D = 90° - Declination_{Polaris}$$

Now, we can project that distance vector on the true North line to obtain the major correction, which is the difference in elevation, along the true North line, between Polaris and the North Pole. That correction depends on LHA_p and is calculated as follows:

$$Corr\left(LHA_p\right) = -D \times \cos(LHA_p)$$

The cosine value in this correction can be either positive or negative, depending whether the LHA_p brings Polaris above or below the true North Pole. The negative sign in the equation comes from the fact that LHA_p is measured from the upper branch of the observer's meridian, while the altitude of Polaris is measured on the lower branch of the same meridian; therefore, the correction is negative whenever $270° < LHA_p < 90°$ (Polaris above the North Pole).

Whenever Polaris is not perfectly on the true North line but offset by an azimuth angle *Zn*, its elevation is measured along the azimuth line from the horizon, which makes the measurement slightly shorter than it would be if measured along the true North line, by an amount proportional to *Zn*, as illustrated in Figures 2 b) and c). It should be noted that *Zn*, also a function of the latitude of the observer, is a small angle, of the order of 1 degree, so *sin(Zn)* is a good approximation of the displacement on the arc of a circle for such a small azimuth deviation.

$$Corr(Zn) = D \times \left|\sin\left(LHA_p\right)\right| \times |\sin(Zn)|$$

That correction is always positive, as seen from Figure 1 b) and c), which accounts for the absolute values in the equation.

The Latitude can then be found by adding the corrections to the measured altitude (*Ho*) of Polaris.

$$Latitude = Ho + Corr\left(LHA_p\right) + Corr(Zn)$$

3.02. The Celestial sphere projection section

This section occupies the central part of the sheet and can be seen either as different views of the earth sphere on which the celestial bodies are projected to their equivalent ground position, or as the celestial sphere on which the earth and observer coordinates are projected. It contains three graphs: the **observer's meridian plane** graph, where the observer zenith projection, his local meridian and the poles lie in the sheet plane; a **top view Zenith projection** represents the circle of equal altitude and the azimuth angle of the observed body, as seen from above the observer's position; a **time diagram** (a top view projection from the south pole, hence west is conterclockwise), that allows to localize the respective meridians (hour angle) of Greenwhich, Ariès, the sun, and the observed celestial body (LHA), with respect to the observer's meridian (M, at top of diagram). It is equivalent to a view over the South pole, showing also the celestial body declination circle. A more detailed description of the individual graphs is given in the next paragraphs.

3.02.01. Observer's meridian plane

This graph of the celestial sphere (or earth) (Figure 3) is the most important to visualize the different sight parameters and the solution of the navigation spherical triangle. In this graph, the observer's assumed zenith position (his AP), makes the vertical, and the circle in the plane of the paper represents the observer's meridian plane, in blue. The observer's projected horizon line becomes the horizontal axis. The poles and the projected equator appear in red, inclined by the value of the co-LAT (90°-LAT) of the observer.

A red line parallel to the equator represents the circle of equal declination (elevation) of the celestial body with respect to the equator as the earth rotates, while the light blue line parallel to the projected horizon represents the circle of equal altitude above the observer's horizon. The red ellipse through the poles is the LHA meridian of the observed celestial body. The intersection of the declination and the LHA gives the position of the celestial body at the time of observation (or its geographical position). This position yields de facto the intersection of the corresponding calculated altitude (light blue line parallel to the horizon) and the azimuth line of the body with respect to the observer's AP (light blue ellipse through the zenith position). The celestial body meridian and the azimuth circle appear as dashed lines when on the hidden side of the sphere.

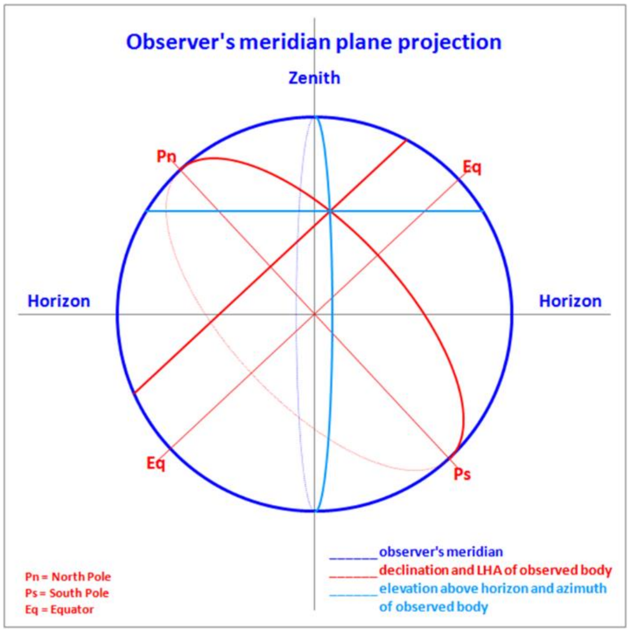


Figure 3. Observer's meridian plane projection.

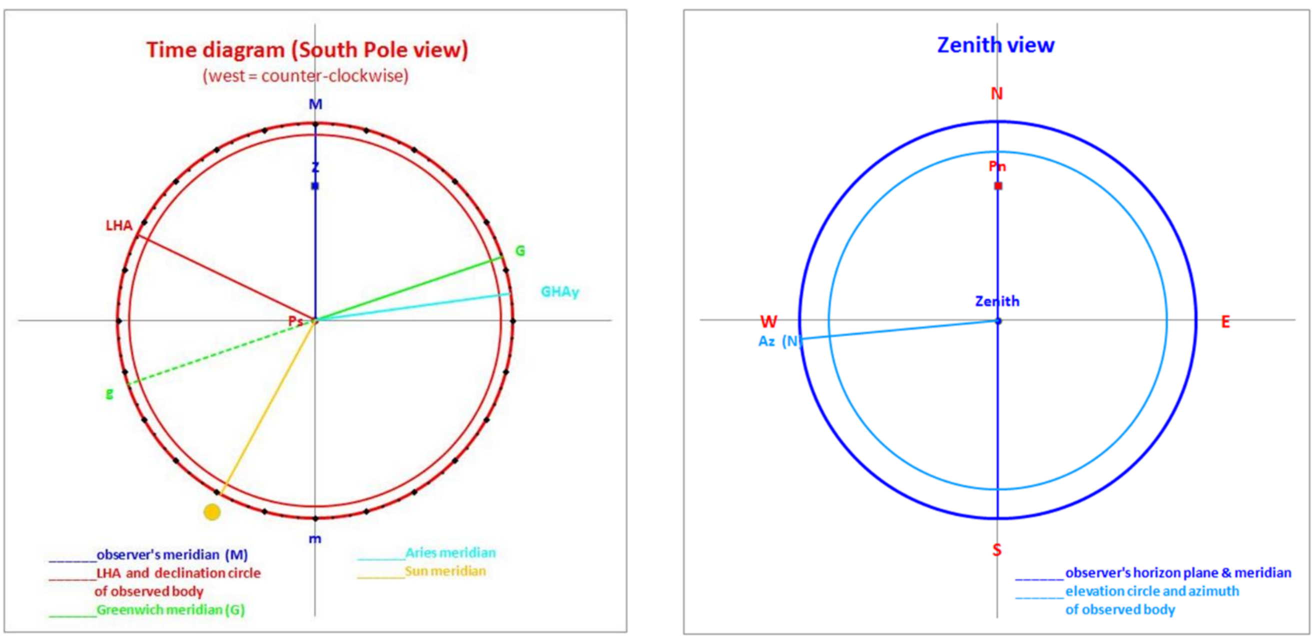


Figure 4. Time diagram (South Pole view) and Zenith view corresponding to observer's meridian plane of Figure 3.

Drawing such a graph from the known celestial body parameters from the *Nautical Almanac*, and using a protractor, one can obtain a crude estimate of the solution to the navigation triangle (and his position), without any calculation.

3.02.02. Time diagram and South Pole View graph

This graph (Figure 4) shows the well known time diagram, which illustrates the hour angles of the various parameters used in sight reduction. It can also be interpreted as a view of the meridians of those parameters, as seen from a position over the top of the South Pole, with west in the counterclockwise direction. It complements the observer's meridian plane graph and illustrates the observer's meridian with the assumed position of the observer on that meridian at the top (M) , the upper and lower branches of the Greenwich meridian (G and g) in green, the meridian of Aries (GHAy) in aqua, the LHA of the celestial body (LHA) with respect to the observer's AP meridian, as well as its declination circle in red, and for the sake of completion, the sun meridian (•) in gold. The declination circle appears dashed when negative (south declination).

For meridian passage sights of the sun, the hour angle of the sun meridian (•) should coincide with that of the observer's meridian (M), at the time of observation, but it may happen that they do not. This is because the sun meridian (•) is the meridian of the **mean sun**, drawn based on the UT observation time, rather than on the real sun position. So, such a time lag (or lead) is a manifestation of the sun equation of time.

3.02.03. Zenith View

Similarly, this graph (Figure 4) shows a view over the observer's zenith and complements the observer's meridian plane graph by showing, in blue, the observer's horizon plane, the observer's meridian, the azimuth line of the body from the observer's AP, and the circle of equal altitude above the horizon corresponding to the calculated altitude (Hc). The North Pole position on the observer's meridian is also shown as a red square. When the altitude is negative (below horizon, the circle appears dashed. This graph is oriented with North at the top.

3.03. The Calculator utilities section

The calculator section of the sheets, shown in Figure 5, lies under the top view Zenith projection and contains various calculator utilities used in previous versions of the software. Their functions are now integrated and performed automatically in the Data input section, but they were left on the sheet as

CONVERTER dd.mm.ss to decimal						
Deg	min	sec				
148	36,8	0				
Decimal:	*148,6133*					
CONVERTER decimal to dd.mm.ss						
Decimal:	0,9983					
Deg	*min*	*sec*				
0	*59*	*53,88*				
GHA INTERPOLATOR for UT min and sec increment						
		deg	min	decimal		
UT 1st hour GHA:		336	11,80	336,1967		
UT 2nd hour GHA:		351	14,20	351,2367		
		min	sec	decimal		
UT min-sec incr:		50	44	0,8456		
		deg	min	decimal		
GHA increment:		*12*	*43,03*	*12,7172*		
DIP-SHORT-OF-HORIZON CALCULATOR						
Height:	ft	10,00				
Dist. to horiz (nmi):		3,60				
DIPshort (min):		*-3,0685*				
ALTITUDE REFRACTION CORRECTION						
		deg	min	decimal		
Altitude (Ha):		16	56,4	16,94		
Correction (min):		*-3,140337*				
TEMPERATURE CONVERSION (°F to °C)						
Temp °F:	59	Temp °C:	15,0			
PRESSURE CONVERSION (inches Hg to mb)						
in Hg:	29,92	mb:	1013,2			
LONGITUDE TO TIME CONVERSION						
Longitude				Time		
Degrees	Minutes	Seconds		Hours	Minutes	Seconds
224	48,25	0		*14*	*59*	*13*
TIME TO LONGITUDE CONVERSION						
Time				Longitude		
Hours	Minutes	Seconds		Degrees	Minutes	Seconds
14	59	13		*224*	*48,25*	
				224	*48*	*15*

Figure 5. Various calculator utilities.

utilities for the user. One can find: a calculator for the *conversion of degrees-minutes-seconds to decimal* values and vice-versa; a *GHA interpolator for the UT minutes-seconds increment*, to interpolate between hourly values in the GHA or other hourly tables; a *dip-short-of-horizon* calculator to evaluate the altitude dip correction when using the shoreline or other proximity surface object as the horizon for a sextant measurement; an *altitude refraction correction* calculator for standard atmospheric conditions (air temp: 10°C; atmospheric pressure: 1010 mbars); a *Fahrenheit to Celcius* temperature converter; a *pressure converter from inches (Hg) to mbars*.

Only the *dip-short-of-horizon* calculator still needs to be used whenever this case is encountered.

3.03.01. Converter degrees-minutes-seconds to decimal

Enter degrees-minutes-seconds as input data, and it is converted to decimal degrees. Input seconds may be ignored and replaced by decimal minutes.

3.03.02. Converter from decimal degrees to degrees-minutes-seconds:

Enter the decimal degrees input value and it is converted to degrees-minutes-seconds.

3.03.03. GHA interpolator for UT min-sec increment

Enter the two consecutive hourly GHA values from the *Nautical Almanac*, bracketing the UT entered as the observation time in the Data input section. Then enter the UT minutes-seconds of the observation time. The interpolated corresponding GHA increment is evaluated in decimal degrees.

3.03.04. Dip-short-of-horizon calculator

If a shoreline or another surface object closer than the horizon is used to make a sextant measurement, then this calculator is the tool to evaluate the dip correction (in minutes). In the calculator, enter the height of the eye in feet or meters and the distance (*dist*) to the shoreline horizon or object in nautical miles (nmi). The dip-short-of-horizon correction is calculated according to the following formula [1]:

$$DIP_{SH} = -60 \times \tan^{-1}\left(\frac{Height_{eye\,(ft)}}{6076.1 \times ds} + \frac{dist_{nmi}}{8268}\right) \quad \text{or}$$

$$DIP_{SH} = -60 \times \tan^{-1}\left(\frac{Height_{eye\,(m)}}{1852 \times ds} + \frac{dist_{nmi}}{8268}\right)$$

It is necessary to use this calculator whenever the situation arises. The calculated correction must be entered manually in the appropriate cell of the Data input section.

3.03.05. Altitude refraction correction calculator

Useful for testing, this calculator performs the same function as in the **Sextant observed altitude block**, in the **Data input section**, using the equation:

$$Alt_{corr} = -\left(\frac{0.97127}{\tan(Ha)} - \frac{0.00137}{\tan^3(Ha)}\right) \text{ for } 15° \le Ha \le 90°$$

$$Alt_{corr} = -\left(\frac{34.133 + 4.197 \cdot Ha + 0.00428 \cdot Ha^2}{1 + 0.505 \cdot Ha + 0.0845 \cdot Ha^2}\right) \text{ for } 0° \le Ha < 15°$$

3.03.06. Temperature conversion (°F to °C)

Converts **Fahrenheit** degrees to **Celsius**.

$$T(°C) = T(°F) \times \frac{5}{9}$$

3.03.07. Pressure conversion (inches of Hg to mbars)

Converts atmospheric pressure data in **inches of mercury (Hg)** to **millibars**.

$$P(mb) = P(in\ Hg) \times 33.863886$$

3.03.08. Longitude to time conversion

Converts longitude difference data to equivalent time difference in sun meridian passage.

3.03.09. Time to longitude conversion

Converts time difference in sun meridan passage to equivalent longitude difference data.

4. The LOP plot worksheet

From the input data in the Sight worksheets and the calculated solution of the navigation spherical triangle (Hc, Ho and AZ_north), the LOP plot worksheet presents automatically a plot of the assumed position (AP), the azimuth line to the observed celestial body for each sight, the intercept (difference between Ho and Hc) and the **circle of position (COP)** for each sight (Figure 6). Traditionally, when the plot is drawn manually, **lines of position (LOP)** are drawn instead of **circles of position**, which are too complicated to calculate and plot. However, **AstronavXls effectively calculates and plots true circles of position (COP)**, centered on the geographical position of the observed celestial body (where the body is observed at zenith). The intersection of two or more circles of position yields the actual position of the observer. The LOP sheet also allows to enter data for **running COPs**, to displace COPs taken at an earlier time, using the speed and course of the boat and elapsed time, in view of establishing the position of the observer at the time of the last sight (running fix).

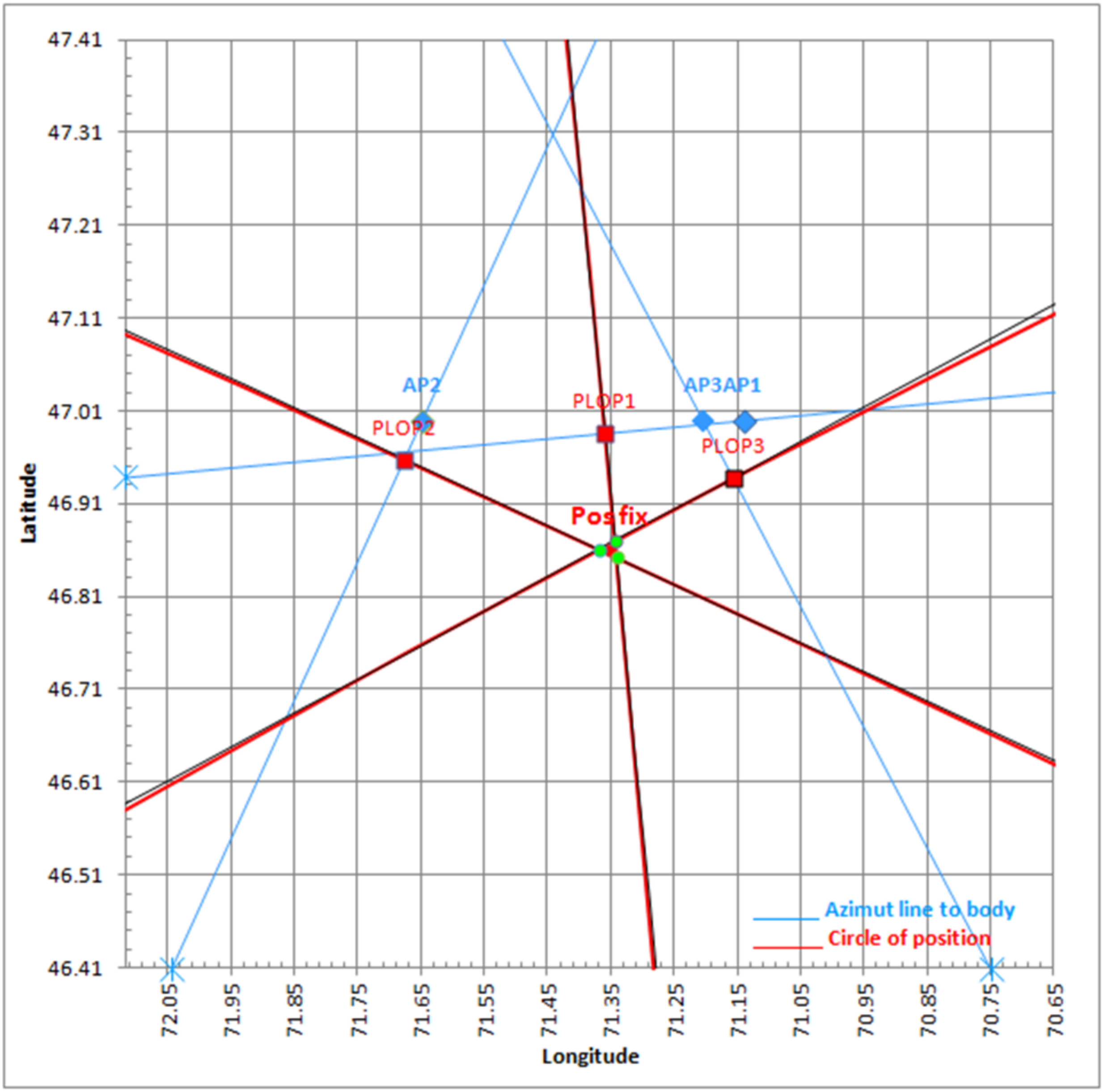


Figure 6. COP chart illustrating azimut lines, COPs and linear bestfit equation.

The graph is updated automatically when new data are entered on the Sight worksheet and no interaction is required normally from the user other than adjusting, if needed and by means of the spin buttons for that purpose, the scale factor of the plot, its center position, the plotted length of the azimuth lines or the COPs, or entering data for a running COP.

The left side of the worksheet shows a note indicating that the longitude scale of the plot is adjusted to account for the assumed latitude, in order to maintain distance and angular conformity (e.g. azimuth direction) and avoid a mapping distortion of the plotted elements. This adjustment is necessary because, on the earth spherical surface, one degree of longitude equals the same distance as one degree of latitude only at the equator; the ratio decreases with the latitude. The **min** and **max** values indicated here are **calculated by a macro** command, from the **AP of the selected sight**, and are used to adjust the X (longitude) and Y (latitude) axis-scales automatically to maintain the desired conformity. The selection of which "sight" will be used for the centering of the chart is useful in the case of a running COP transported to the time of the last sight for a fix; it would be appropriate to select the last sight in such a case.

Here follows the description of the parameters and components appearing on the "LOP plot" worksheet.

Scale factor: the number entered as the scale factor allows the user to change the span of the plot (effectively zooming the chart in or out). The number translates to degrees shown on the plot on each side of the AP whole degrees of latitude. The longitude scale is adjusted accordingly to maintain the conformity of the map. The numbers below the scale factor are intermediate values used for the automatic adjustment of the longitude axis. The chart is automatically updated every time there is a change on the sheet, but clicking on the chart will also result in an update.

COP chart:

in blue: on that chart (Figure 6), the AP and the azimuth lines are marked in blue for each sight. A star appears on the edge of the graph at the end of the azimuth line (**this is not the observed body geographical position**) to indicate which side is "toward" the celestial body.

The extent of each azimuth line can be modified, if needed, by changing the *length* parameter, near the azimuth, in the corresponding sight data on the sheet, underneath the chart. This parameter determines the extension of the azimuth line on each side of the AP, in terms of degrees.

in red: the intersection of the COP and the azimuth line are marked by a red square, and a **true circle of position (COP)** is plotted, centered on the celestial body geographical position (where the body appears at zenith). The COP is obtained by rotating a great circle arc, of length corresponding to the measured co-altitude of the celestial body, around the ground position of the celestial body and on each side of the azimuth line, using the great-circle equations. Hence, the COPs are exact plots, not approximations. Since true COPs are plotted, there is no need to use offset tables to correct for curvature when the intersection with other COPs or the nearest dead reckoning position is some distance away from the intercept with the azimuth line.

The extent of each COP can be modified, when required, by changing the *increment in azimuth (dZ or dimCOP#)* parameter at the bottom of the page, under the item **Plot of body# circle of position (COP#)**. This parameter determines the azimuth step increase on each side of the azimuth line, in degrees, for the plotting of the circle of position.

Chart-center offset: at the bottom left of the LOP chart, there are spin buttons (Figure 7) used to conveniently displace the center of the chart left, right, up or down, in order to find the best view of the COP intersection region. The displacement is proportional to the **scale**

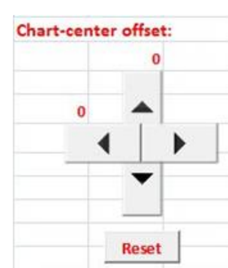


Figure 7. Chart offset spin buttons.

factor defined above.

Assumed position: under this item, below the chart, several useful data for each COP plot, copied from their respective Sight sheets, including data input cells to account for a **running COP**.

Running COP: each COP plot can be made a running COP (moving the COP taken at an earlier time to a later time by dead reckoning) by entering the boat speed and course, and the elapsed time since the sextant observation was done. The AP is moved to the new position calculated from the speed, course and time elapsed, and takes the azimuth line and the plotted COP with it (the COP is not recalculated).

Calc. Altitude: under this item, other parameters from the input data sheet are repeated, such as the **altitude difference,** in degrees, between the observed and computed altitudes Ho and Hc, the **distance from AP to COP** (intercept value) on the azimuth line, the **distance from to the geographic position of the celestial body,** from the COP (radius of the circle of position), and the coordinates of the **position of the COP** on the azimuth line.

Position of COP on azimuth line: the decimal coordinates of the intersection of the circle of position and the azimuth line, taking into account the displacement of a running COP whenever it is the case.

Note: in the case of **COP1**, whenever the **meridian sight check box** is checked in the **"Sight1" input data sheet**, the GHA of the body at meridian crossing is converted to longitude and replaces the **Assumed Position longitude data** for **sight1**. Therefore, in such a case, the **position of COP1 on the azimuth line** indicates the **observer's position** and is transferred to the **Meridian passage sight position** **green area**, to the right of the graph. This applies only to "**Sight1"** data sheet, as there is no meridian crossing check box in the other sight input data sheets. The accuracy of the longitude coordinate in such a case is subject to caution.

Find intersection of COPs by linear bestfit approximation: on the right side of the chart appears **three checkboxes**, one for each COP (Figure 8). A **fourth checkbox** allows the display of the linear interpolation lines (in black) on the graph. When two, or more, COPs are checked as available, a macro command is launched to find a linear best fit equation for each COP and calculate a fix position from the intersection(s) (Figure 8). Since the COPs are individual vector plots of circular lines of position, there are no equations that describe them in the polar coordinates of the terrestrial sphere. The macro, through an iterative process, finds a linear approximation equation to the curve **in the vicinity of the intersection point and tangential to it**, which helps finding the intersection of two COPs with very high accuracy. Once two COPs have been estimated, the coordinates of their intersection point appears in the appropriate intersection description and a corresponding **green dot** is shown on the chart. When the intersections of three COPs are estimated, a **three-points** fix solution is presented and shown as a **red dot**. The following describes the procedure to find the intersection of COPs.

Note: the extension of each COP must be such as the COP curves plotted do cross each other in order for the macro command to be able to calculate the intersection.

***The individual curves may be extended by modifying the dimCOP# parameter at the bottom of the page, under the item* Plot of body# circle of position (COP#).**

1. For each sight or COP available, click the corresponding checkbox to activate a macro that finds the linear best fit equation (in the vicinity of the azimuth line if only one checkbox is checked), from the COP plot data found in the corresponding "CALC" worksheet. A fourth check box allows the user to select whether he wants to see or not the linear approximations lines on the graph.

2. When **2 COPs have been checked**, the macro starts an **iterative process** to find the intersection. The starting linear estimate is calculated tangential to the central position of each COP, where it crosses the azimuth line. The intersection of the two linear approximations is then used to find the closest point on each COP, and a new linear approximation is made tangential to that closest point to find the new intersection point. Thus, the process converges until the new calculated intersection point moves minimally from the previous one, which stops the iterative process. The last calculated intersection is then very close to the true intersection point, as can be observed on the graph by means of the black lines drawn to illustrate the last linear approximation equations (when the checkbox to that purpose is checked), and **the green dot** for the calculated intersection. The intersection coordinates appear in the appropriate intersection description, and the slope and offset of each approximation equation is recorded in the cells adjacent to the checkboxes.

Find intersection of COPs by linear bestfit approximation
n=0: x=my+b n=1: y=mx+b
Click on available COPs

		m	b	n
☑ Check if COP1 is available	LOP1:	0,138647153	64,84557399	0
☑ Check if COP2 is available	LOP2:	0,315246665	24,360803	1
☑ Check if COP3 is available	LOP3:	-0,36655749	73,02052891	1
☑ Check to view linear approximations				

	deg	min	decimal
Intersection COP1-COP2			
LAT	N 46	51,06	46,8510
LON	W 71	20,49	71,3415
Intersection COP1-COP3			
LAT	N 46	52,12	46,8686
LON	W 71	20,63	71,3439
Intersection COP2-COP3			
LAT	N 46	51,57	46,8596
LON	W 71	22,13	71,3688

Three-points fix solution (3 COPs required)			
LAT	N 46	51,583	46,8597
LON	W 71	21,084	71,3514

Meridian passage sight position ("Sight1")			
LAT	N 0	00,000	
LON	W 0	00,000	
Meridian passage sight position ("Sight2")			
LAT	N 0	00,000	
LON	W 0	00,000	
Meridian passage sight position ("Sight3")			
LAT	N 0	00,000	
LON	W 0	00,000	

☐ Check to average the position coordinates of several meridian passage sights

Average meridian passage position ("Sights 1, 2, 3")			
LAT	N 0	0,000	
LON	W 0	0,000	

Figure 8. Best fit linear equation parameters in the intersection vicinity and computed intersections.

3. **If three COPs are available**, the **three-points** geometrical average solution appears as the best position fix, in the bottom section, and is shown as a **red dot** on the chart. **That's where you should be**!!!...

Meridian passage sight position: whenever the **green checkbox** at the bottom of the data block in **Sight** sheets is checked, this **green area** shows the coordinates of the observer at the time of passage of a celestial body over the observer's meridian (e.g. a midday sight of the sun). The longitude data is obtained by converting the **GHA of the body** at the time of transit as the **new Assumed Position Longitude** on the **Sight** sheet. For a single transit sight, the longitude data is subject to caution in its accuracy as it is difficult to estimate the exact time of meridian crossing, that is when the celestial body reaches its maximum altitude above the horizon. But the accuracy of the longitude coordinate may be improved by averaging coordinates for a number of such sights made at the same observer's position. Up to three sights can be averaged by checking the check box to that effect.

Enjoy your sextant!

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