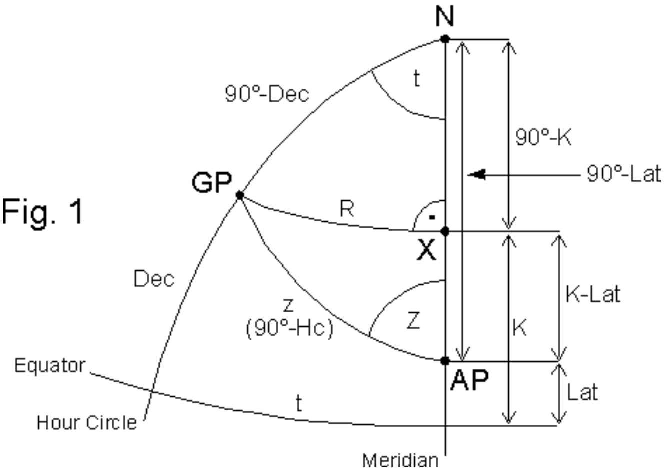
The Ageton Tables

by Henning Umland

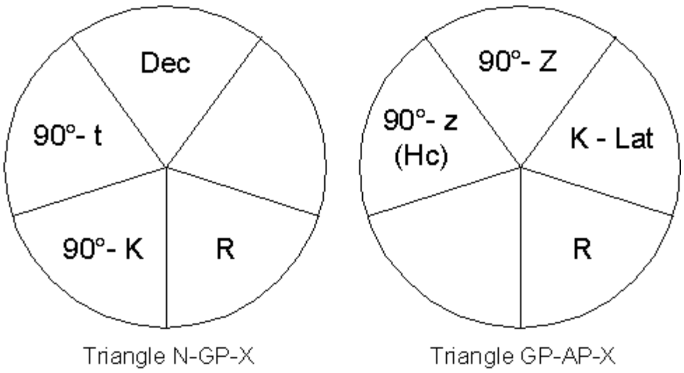
The sight reduction tables developed by Arthur A. Ageton are based upon the divided navigational triangle. The latter is derived from the general (oblique) navigational triangle GP-N-AP by constructing a great circle passing through GP and intersecting the local meridian (the meridian going through AP) at a right angle (*Fig. 1*):

Fig. 1



N is the north pole. AP is the observer's assumed (or estimated) position, and GP the geographic position of the observed body. t is the meridian angle. The first right triangle is formed by N, GP, and X, the second one by GP, AP, and X. K is the angular distance of X from the equator, measured through AP. The **auxiliary parts** R and K are intermediate quantities used to calculate z (or Hc) and Az. Both triangles are solved using **Napier's Rules of Circular Parts** (see mathematical literature). *Fig. 2* illustrates the corresponding circular diagrams:

Fig. 2



According to Napier's rules, Hc and Z are calculated by means of the following formulas. Observe the correct sign of each quantity (northern Lat or Dec: positive, southern Lat or Dec: negative, western t: positive, eastern t: negative).

$$\sin R = \sin t \cdot \cos Dec \qquad \Rightarrow \qquad R = \arcsin\left(\sin t \cdot \cos Dec\right)$$

$$\sin Dec = \cos R \cdot \sin K \qquad \Rightarrow \qquad \sin K = \frac{\sin Dec}{\cos R} \qquad \Rightarrow \qquad K = \arcsin \frac{\sin Dec}{\cos R}$$

Substitute 180°–K for K in the following formula if $|t| > 90°$.

$$\sin Hc = \cos R \cdot \cos\left(K - Lat\right) \qquad \Rightarrow \qquad Hc = \arcsin\left[\cos R \cdot \cos\left(K - Lat\right)\right]$$

$$\sin R = \cos Hc \cdot \sin Z \qquad \Rightarrow \qquad \sin Z = \frac{\sin R}{\cos Hc} \qquad \Rightarrow \qquad Z = \arcsin \frac{\sin R}{\cos Hc}$$

For further calculations, substitute 180°–Z for Z if K and Lat have opposite signs or if $|K| < |Lat|$.

To obtain the **true azimuth**, $\mathbf{Z_N}$ (0°... 360°), the following **rules** have to be applied:

$$Z_N = \begin{cases} -Z & \text{if} \quad Lat > 0\,(\mathrm{N}) \text{ AND } t < 0\,(\mathrm{E}) \\ 360° - Z & \text{if} \quad Lat > 0\,(\mathrm{N}) \text{ AND } t > 0\,(\mathrm{W}) \\ 180° + Z & \text{if} \quad Lat < 0\,(\mathrm{S}) \end{cases}$$

Using the secant and cosecant function ($\sec\alpha = 1/\cos\alpha$, $\csc\alpha = 1/\sin\alpha$), the equations for the divided navigational triangle are stated as:

$$\csc R = \csc t \cdot \sec Dec$$

$$\csc K = \frac{\csc Dec}{\sec R}$$

$$\csc Hc = \sec R \cdot \sec\,(K - Lat)$$

$$\csc Z = \frac{\csc R}{\sec Hc}$$

In logarithmic form, these equations are stated as:

$$\log\csc R = \log\csc t + \log\sec Dec$$

$$\log\csc K = \log\csc Dec - \log\sec R$$

$$\log\csc Hc = \log\sec R + \log\sec\,(K - Lat)$$

$$\log\csc Z = \log\csc R - \log\sec Hc$$

Having the logarithms of the secants and cosecants of angles available in the form of a suitable table, we can solve a sight by a sequence of simple additions and subtractions (beside converting the angles to their corresponding log secants and log cosecants and vice versa). Apart from the table itself, the only tools required are a sheet of paper and a pencil.

The Ageton Tables (H.O. 211), first published in 1931, are based upon the above formulas and provide a very efficient arrangement of angles and their so-called A and B functions:

$$A(\alpha) = 100000 \cdot \log\csc\alpha$$

$$B(\alpha) = 100000 \cdot \log\sec\alpha$$

The factor 100,000 is used to obtain A and B values without leading zeros.

Since logarithms of negative numbers do not exist, one has to calculate with absolute values when using the Ageton Tables. In order to obtain correct results, the procedure includes certain rules which have to be observed.

In most cases, the accuracy of Ageton's tables is sufficient without interpolaton, which makes their use very convenient .

The instructions for using the Ageton Tables are:

Mark each angle (Lat, Dec, and t) with its appropriate name (N, S, W, E). Treat all angles as absolute values (ignore signs):

Positive Lat or Dec	↔	N
Negative Lat or Dec	↔	S
Positive t	↔	W
Negative t	↔	E

Enter the table with the meridian angle, t, calculated from GHA and the assumed or estimated longitude, and note A(t).
Enter the table with the declination, Dec, and note A(Dec) and B(Dec).
Calculate the A value of R, A(R):

$$A(R) = A(t) + B(Dec)$$

Find the tabulated A value nearest to A(R) and note the corresponding value for B(R). The angle R itself is not needed.

Calculate the A value of K, A(K):

$$A(K) = A(Dec) - B(R)$$

Find the tabulated A value nearest to A(K) and note the corresponding angle K. **When t is greater than 90°, take K from the bottom of the table (180°–K), otherwise from the top. Give K the same name as Dec (N or S)**.

Combine K and the assumed or estimated latitude, Lat. Form the sum of both angles if K and Lat are of contrary name. Subtract the smaller from the greater angle if K and Lat have the same name. Enter the table with the angle thus obtained and find the corresponding B value, B(K~Lat).

Calculate A(Hc), the A value of the computed altitude:

$$A(Hc) = B(R) + B(K \sim Lat)$$

Find the tabulated A value nearest to A(Hc), and note the corresponding B value, B(Hc), and **Hc, the computed altitude**.

Calculate the A value of the azimuth angle Z, A(Z):

$$A(Z) = A(R) - B(Hc)$$

Find the tabulated A value nearest to A(Z) and note **Z,** the corresponding **azimuth angle**. **Always take Z from the bottom of the table, except when K is same name and greater than Lat (including Lat=0), in which case take Z from the top of the table.**

To obtain the **true azimuth**, $\mathbf{Z_N}$, apply the following conversion rules:

$$Z_N = \begin{cases} Z & \text{if} \quad Lat > 0\,\text{(N)} \;\; \text{AND} \;\; t < 0\,\text{(E)} \\ 180° - Z & \text{if} \quad Lat < 0\,\text{(S)} \;\; \text{AND} \;\; t < 0\,\text{(E)} \\ 180° + Z & \text{if} \quad Lat < 0\,\text{(S)} \;\; \text{AND} \;\; t > 0\,\text{(W)} \\ 360° - Z & \text{if} \quad Lat > 0\,\text{(N)} \;\; \text{AND} \;\; t > 0\,\text{(W)} \end{cases}$$

If the observer is exactly on the equator (Lat=0), the rules can still be used when substituting Dec for Lat.

These conversion rules for the tables differ from those stated earlier because the Ageton Tables are based on absolute values.

The Ageton method has a few weak points which require the user's attention:

If a body is near the visible horizon, it may be below the celestial horizon due to the effects of dip and refraction, and the altitude may be negative. Hc is negative if K is of the same name as Lat and greater than 90°+Lat, or if K is of contrary name to Lat and greater than 90°–Lat. In the latter case, Z is less than 90° and should be taken from the top of the table if K is greater than 180°–Lat.

If the azimuth angle, Z, is approximately 90°, the A value of Z may be a negative quantity if the tables have been used without interpolation (which is usually the case). This can be avoided by interpolating throughout or by substituting the nearest tabulated A value for the calculated A value of R, A(R), at the top of the azimuth column.

The Ageton method may give inaccurate results if t or K is near 90°. Therefore, values between 82° and 98° should be avoided and the respective sight discarded (forbidden range). The accuracy in this range can be improved by interpolating B(R) from A(R) or using the Ageton tables in combination with Sadler's sight reduction technique*.

* See A. E. Bayless, Compact Sight Reduction Table (modified H. O. 211 Table), Cornell Maritime Press

The attached tables are a modified, more accurate version of the original Ageton Tables. The log secants and log cosecants have been multiplied by 10^6 instead of 10^5. Angles are tabulated to the nearest 0.2' (0.5' in the original tables). Each table comprises the log secants and log cosecants of an interval of 1° (original tables: 2.5°). A and B values have been calculated with a spreadsheet program. The last digit has been rounded.

The first row of each table shows an integer number marking the beginning of the respective one degree interval. The latter is divided into six subintervals of 10 arcminutes each (second row). Below each subinterval are two columns containing the A and B values for the intermediate angles which appear, tabulated to 0.2', in the leftmost column of the table. The corresponding supplementary angles can be read from the two bottom lines and the rightmost column of the table.

The tabulated numbers in the following examples refer to the modified tables (0.2' intervals), **not** to Ageton's original tables:

Example 1

Lat –30° 10' → 30° 10' S
Dec +20° 08' → 20° 08' N
t +45° 33' → 45° 33' W

A(t) = A(45° 33') = 146386
A(Dec) = A(20° 08') = 463182
B(Dec) = B(20° 08') = 27383

A(R) = A(t) + B(Dec) = 146386 + 27383 = 173769

The tabulated A value nearest to A(R) is 173761. The corresponding value for B(R) is 129519.

A(K) = A(Dec) – B(R) = 463182 – 129519 = 333663.

The tabulated A value nearest to A(K) is 333658. The corresponding value for K is 27° 38.0' (taken from the top of the table since t < 90°).The name of K is N (same as Dec).

Since K and Lat are of contrary name, we form the sum of both quantities:

K + Lat = 27° 38.0' + 30° 10.0' = 57° 48.0'

B(K+Lat) = 273374

A(Hc) = B(R) + B(K+Lat) = 129519 + 273374 = 402893

The tabulated A value nearest to A(Hc) is 402921.

Hc = 23° 17.6'.

The corresponding value for B(Hc) is 36924

A(Z) = A(R) – B(Hc) = 173769 – 36924 = 136845

The tabulated A value nearest to A(Z) is 136841.

Since K (N) and Lat (S) are of contrary name, we take Z from the bottom of the table: Z = 133° 08.2'.

Since Lat < 0 and t > 0 we have to add 180° to obtain Z_N:

Z_N = 313° 08.2'

The exact values calculated with the formulas for the oblique spherical triangle are:

Hc = 23° 17.8' Z_N = 313° 08.2'.

Example 2

Lat	–70° 15'	→	70° 15' S
Dec	–50° 22'	→	50° 22' S
t	–110° 03'	→	110° 03' E

A(t) = A(110° 03') = 27152 (Enter t at bottom of table since t > 90°.)
A(Dec) = A(50° 22') = 113429
B(Dec) = B(50° 22') = 195266

A(R) = A(t) + B(Dec) = 27152 + 195266 = 222418

The tabulated A value nearest to A(R) is 222421. The corresponding value for B(R) is 96589.

A(K) = A(Dec) – B(R) = 113429 – 96589 = 16840 .

The tabulated A value nearest to A(K) is 16841. The corresponding value for K is 105° 51.2' (taken from the bottom of the table since t > 90°).The name of K is S (same as Dec).

Since K and Lat are of same name, we form the difference of both quantities:

K – Lat = 105° 51.2' – 70° 15.0' = 35° 36.2'

B(K–Lat) = 89874

A(Hc) = B(R) + B(K–Lat) = 96589 + 89874 = 186463

The tabulated A value nearest to A(Hc) is 186452.

Hc = 40° 36.8'.

The corresponding value for B(Hc) is 119690.

A(Z) = A(R) – B(Hc) = 222418 – 119690 = 102728

The tabulated A value nearest to A(Z) is 102719.

Since K (S) and Lat (S) are of same name and K greater than Lat, we take Z from the top of the table:

Z = 52° 07.6'.

Since Lat < 0 and t < 0, we have to subtract Z from 180° to obtain Z_N:

Z_N = 127° 52.4'

The exact values calculated with the formulas for the oblique spherical triangle are:

Hc = 40° 36.8' Z_N = 127° 52.5'.

Example 3

This example demonstrates how the accuracy may be affected when t (or K) is in the forbidden range.

Lat +53° 10' → 53° 10' N
Dec +38° 06' → 38° 06' N
t –90° 54' → 90° 54' E

A(t) = A(90° 54') = 53.58 (Enter t at bottom of table since t > 90°.)
A(Dec) = A(38° 06') = 209690
B(Dec) = B(38° 06') = 104061

A(R) = A(t) + B(Dec) = 53.58 + 104061 = 104114.58

The tabulated A value nearest to A(R) is 104121. The corresponding value for B(R) is 209593.

A(K) = A(Dec) – B(R) = 209690 – 209593 = 97

The tabulated A value nearest to A(K) is 96.85. The corresponding value for K is 91° 12.6' (taken from the bottom of the table since t > 90°).The name of K is N (same as Dec).

Since K and Lat are of same name, we form the difference of both quantities:

K – Lat = 91° 12.6' – 53° 10.0' = 38° 02.6'

B(K–Lat) = 103725

A(Hc) = B(R) + B(K–Lat) = 209593 + 103725 = 313318

The tabulated A value nearest to A(Hc) is 313337.

Hc = 29° 04.8'.

The corresponding value for B(Hc) is 58517.

A(Z) = A(R) – B(Hc) = 104114.58 – 58517 = 45597.58

The tabulated A value nearest to A(Z) is 45604.
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Since K (N) and Lat (N) are of same name and K greater than Lat, we take Z from the top of the table:

Z = 64° 12.0'.

Since Lat > 0 and t < 0, we leave Z unchanged:

Z_N = 64° 12.0'

The exact values calculated with the formulas for the oblique spherical triangle are:

Hc = 29° 06.5' Z_N = 64° 14.0'

Thus, the error in altitude is –1.7', and the error in azimuth is –2.0'.

Errors occurring in the forbidden range are hard to predict and can be more dramatic than shown here.

In order to improve the results of example 3, we interpolate B(R) from A(R) and repeat the calculations with the value thus obtained (tabulated values printed in bold face):

A	B
104121	**209593**
104114.58	*X*
104101	**209625**

$$\mathrm{B(R)} = X = 209625 + \frac{104114.58 - 104101}{104121 - 104101} \cdot \left(209593 - 209625\right) = 209625 - 21.728 = 209603.272$$

A(K) = A(Dec) – B(R) = 209690 – 209603.272 = 86.728

The tabulated A value nearest to A(K) is 86.98. The corresponding value for K is 91° 08.8' (taken from the bottom of the table since t > 90°).The name of K is N (same as Dec).

Since K and Lat are of same name, we form the difference of both quantities:

K – Lat = 91° 08.8' – 53° 10.0' = 37° 58.8'

B(K–Lat) = 103349

A(Hc) = B(R) + B(K–Lat) = 209603.272 +103349 = 312952.272

The tabulated A value nearest to A(Hc) is 312973.

Hc = 29° 06.4'.

The corresponding value for B(Hc) is 58630.

A(Z) = A(R) – B(Hc) = 104114.58 – 58630 = 45484.58

The tabulated A value nearest to A(Z) is 45482.

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Since K (N) and Lat (N) are of same name and K greater than Lat, we take Z from the top of the table.

Z = 64° 14.0'.

Since Lat > 0 and t < 0, we leave Z unchanged:

Z_N = 64° 14.0'